Construction of full black-hole binary waveforms for LISA

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Results from
MH, S. Husa, J. A. González, U. Sperhake, B. Brügmann, PRD**77**, 044020 (2008),
A. Gopakumar, MH, S. Husa, B. Brügmann, arXiv:0712.3737,
MH, S. Husa, B. Brügmann, A. Gopakumar, arXiv:0712.3787
and
some new results
(With the NR groups at AEI and Jena, and the AEI data analysis group)

Purpose of this talk

LISA data analysis requires full waveforms

- Current search templates for ground-based detectors are based on PN theory
- PN waveforms are "cut off" before merger
  - (Or an educated guess is made for the merger waveform (e.g., EOB)
- The merger waveform significantly improves LISA parameter estimation
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Producing complete waveforms

- Use numerical relativity (NR) simulations to get the merger waveform.
- However, these cannot cover more than ~ 10 cycles before merger.
- One solution is to connect PN and NR waveforms to get complete waveforms.
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- But are the PN and NR waveforms accurate enough?

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The main points of this talk

- NR waveforms are very accurate – and quickly getting *much* more accurate.
- Waveforms from different groups and codes agree extremely well.
- The NR waveforms can quantify the accuracy of PN waveforms

NR waveforms are very accurate – and getting better!

Example: Equal-mass, nonspinning black-hole binaries.

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October, 2007 (5 months later):

- Cornell-Caltech spectral code: 30 cycles before merger, overall phase error: $<$ **1/500 cycles**.

By the time LISA flies, equally (or more) accurate full GR waveforms will exist for a wide subsection of parameter space.

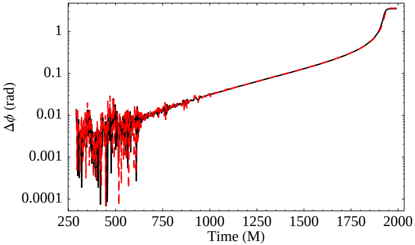
Accuracy of NR waveforms

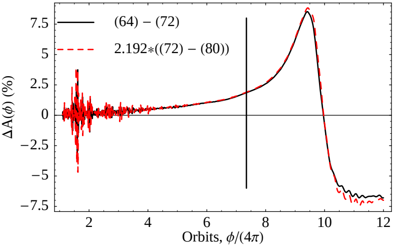
The BAM code uses

- moving-box mesh refinement
- 6th-order finite differencing in the bulk

For an equal-mass nonspinning binary, with 18 cycles before merger,

- Both amplitude and phase show good 6th-order convergence.
- Extrapolate with respect to resolution *and* radiation extraction radius
- The uncertainty in the extrpolated phase is about $1/50$ cycles.
- The unceratinty in the extrapolated amplitude is less than 2%.





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A project is underway to formally compare the last $1000M$ of more recent equal-mass nonspinning waveforms

Informal comparisons already suggest *excellent* agreement

- Amplitudes agree within a few percent
- Phases agree within $1/10$ cycles
- This is within the error bars quoted for all the results

Numerical waveforms are accurate and robust

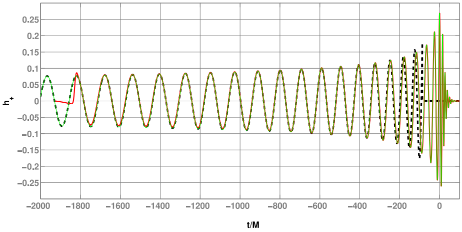
Current long NR waveforms

"Long" waveforms means more than 15 cycles before merger.

With the BAM code, we have produced long waveforms for

- Equal-mass nonspinning binaries
- Unequal-mass nonspinning binaries for $M_1/M_2 \leq 4$.
- Equal-mass (non-precessing) spinning binaries $|S_i/M_i^2| \leq 0.85$.

Some other long waveforms have been produced by other groups (Other spin configurations [RIT], eccentric orbits [PSU])



Achieving non-eccentric inspiral

We don't just need an accurate simulation – we need accurate initial physical parameters.

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- Integrate PN equations of motion for 1000s of orbits up to starting separation
- Read off momenta at starting separation, and use as initial parameters in simulation
- Works extremely well for nonspinning binaries: $e \sim 0.002$.
- For spinning binaries, eccentricity is too high, $e > 0.01$ for some cases.
- Use PN integration to calculate modification Δp necessary to give $\Delta e \sim e$.
- One iteration of this procedure reduces eccentricity to $e \leq 0.003$.

PN-NR comparison: spinning binaries

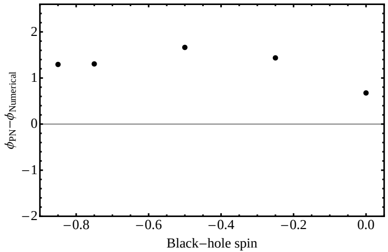
Equal mass, equal spins, *parallel* or *anti-parallel* to orbital angular momentum. (Results for *anti-parallel* simulations are preliminary)

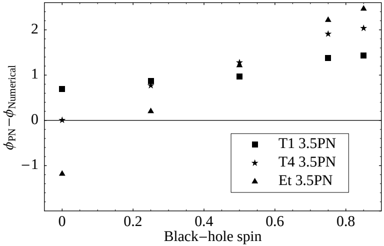
Consider $|S_i/M_i^2| = 0, 0.25, 0.5, 0.75, 0.85$.

Measure the phase disagreement over the 10 cycles up to $M\omega = 0.1$.

Look at the 3.5PN case (even though not all spin terms are known beyond 2.5PN).

- For all approximants, phase disagreement is less than 2.5 radians.





The last slide

Summary

- NR waveforms are accurate
  - $\Delta\phi < 1/50$ cycles
  - $\Delta A/A < 0.02$.

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- NR waveforms agree well between groups and codes
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Usefulness of NR waveforms

- Produce complete waveforms by making PN-NR hybrids. (Sascha's talk on Thursday.)
- The merger waveforms can drastically improve LISA parameter estimations. (Stas's talk on Thursday.)