

Simulations of Extreme-Mass-Ratio Inspirals Using Pseudospectral Methods



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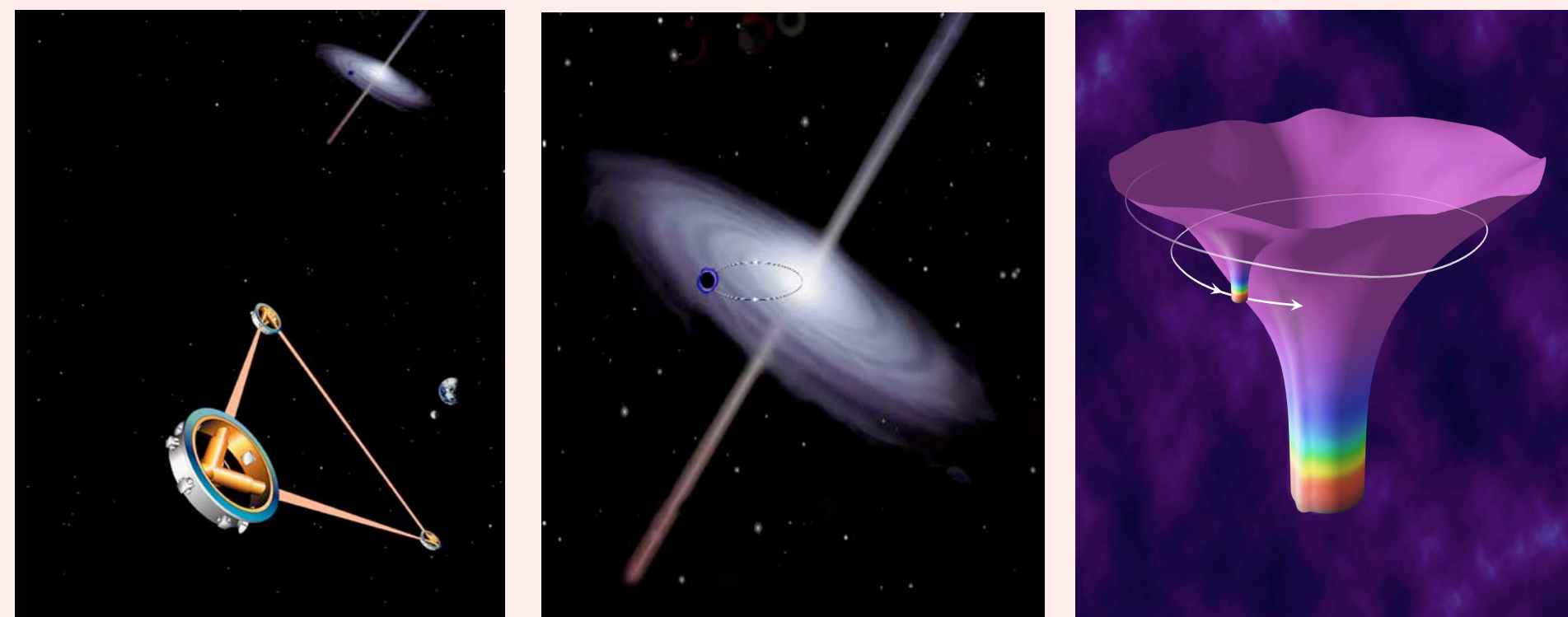
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Extreme-mass-ratio inspirals (EMRIs), stellar-mass compact objects inspiraling into a massive black hole, are one of the main sources of gravitational waves expected for the Laser Interferometer Space Antenna (LISA) observatory. To extract the EMRI signals from the LISA data stream we need very accurate theoretical templates of the gravitational waves that they produce. In order to do so, we need to consider the gravitational back-reaction, that is, how the gravitational field of the stellar-mass compact object affects its own trajectory. In general relativity, the back-reaction is described in terms of a gravitational 'self-force', and the foundations to compute it have been laid several years ago. However, parts of the calculation of the self-force have to be performed numerically. In this poster we describe an effort towards the computation of the self-force based on time-domain pseudospectral methods, in which we try to take advantage of the high precision that this method provides.

Main Motivation

To have a theoretical understanding of astrophysical sources of Gravitational Waves that the future NASA/ESA mission LISA [1] will be able to observe. Simulations of sources like Extreme-Mass Ratio Binaries (EMRIs) can provide key information for the development of techniques of Gravitational Wave Astronomy.



EMRIs are astrophysical systems consisting of a stellar-mass object, $m \sim (1-10) M_\odot$ orbiting a Super Massive Black Hole (SMBH), $M \sim (10^3-10^8) M_\odot$. Hence the mass ratios ($\mu = m/M$) are in the range $10^{-2} - 10^{-8}$. EMRIs spend around $10^5 - 10^6$ cycles inside the LISA band during the last year before plunge. In the LISA band they lose energy through the emission of Gravitational Waves, and their orbits shrink until they merge. The waves will be emitted at a high Signal-to-Noise ratio, which will allow us to extract crucial astrophysical information.

The Computational Challenge

- ◆ The steps to describe EMRIs with relativistic perturbation theory are the following:
 1. Compute the perturbation created by the stellar-type object in the spacetime geometry of the MBH.
 2. Estimate how these perturbations affect the orbital motion of the stellar-type object: the radiation-reaction effect.
 3. Compute the waveforms generated in the inspiral.
- ◆ In order to perform numerical simulations of EMRIs we have to resolve very **different spatial scales** (the size of the stellar object, the Black Hole size, the wavelengths of the gravitational radiation) and also very **different temporal scales** (the orbital period, the radiation reaction timescale). This fact suggests that we need to use spatial and temporal grid adaptivity.
- ◆ Time-domain simulations of EMRIs may have to face stability problems and the appearance of spurious high frequency modes.

The Numerical Setup

The **Pseudo-Spectral Collocation (PSC)** method is a very precise technique to solve partial differential equations (PDEs). Due to the accuracy that EMRIs calculations for LISA require, the PSC method is a convenient option to describe this kind of systems.

In the PSC method the solution of the problem is approximated by a spectral decomposition of the type:

$$\Phi_N(t, x) = \sum_{k=0}^N a_k(t) \phi_k(x)$$

where $\{\phi_k\}$ is a set of basis functions and $\{a_k\}$ are the spectral components of the expansion.

In the PSC method, the coefficients of the spectral expansion are found by imposing that the equation is identically satisfied at a set of N **collocation points**, usually associated with the basis functions.

We use Chebyshev polynomials as basis functions and the Chebyshev-Lobatto collocation points.

Calculations for a charged Particle orbiting a non-rotating Black Hole. I

Our goal is, using spectral methods, to solve the equations describing the scalar field generated by a charged particle orbiting a nonrotating black hole

$$ds^2 = f(-dt^2 + dr^{*2}) + r^2 d\Omega^2 \quad r^* = r + 2M \ln\left(\frac{r}{2M} - 1\right)$$

$$f = 1 - \frac{2M}{r}, \quad d\Omega^2 = d\theta^2 + \sin^2 \theta d\varphi^2$$

and then, to calculate the particle self-force that will produce the inspiral into the black hole.

Calculations for a charged Particle orbiting a non-rotating Black Hole. II

We have to solve the scalar wave equation:

$$g^{\alpha\beta} \nabla_\alpha \nabla_\beta \Phi(x) = -4\pi \rho(x)$$

with the particle energy density given by $\rho(x) = q \int_\gamma \delta_4(x, z(\tau)) d\tau$ and $\Phi(x)$ is the scalar dynamical gravitational field.

The scalar particle has a charge q , and moves along the world line γ parametrized by the proper time τ .

◆ As a first step we have to compute the field generated by the particle moving in a geodesic of the Schwarzschild spacetime.

Expanding the scalar field in spherical harmonics

$$\Phi(x) = \sum_{l=0}^{\infty} \sum_{m=-l}^l \frac{1}{r} \psi_{lm}(t, r) Y^{lm}(\theta, \varphi)$$

we obtain, for each harmonic component, $l+1$ wave equations of the form:

$$(-\partial_t^2 + \partial_r^{*2} - V_l) \psi_{lm} - S_{lm} = 0$$

where the singular source term is

$$S_{lm} = A \delta[r^* - r_p^*(t)], \quad A = -4\pi q \frac{f(r_p)}{r_p E} \bar{Y}_{lm}\left(\frac{\pi}{2}, \varphi_p\right)$$

and the black hole potential is given by the Regge-Wheeler potential for scalar fields

$$V_l = f \left(\frac{2M}{r^3} + \frac{l(l+1)}{r^2} \right) \quad f = 1 - \frac{2M}{r}$$

Self-Force on a Charged Particle

The second step in order to obtain the self-force is to compute the modes of the field gradient. The naive expression for the self-force is given by

$$F_\alpha(\tau) = q \nabla_\alpha \Phi(z(\tau))$$

However, due to the particle singularity, the self-force calculated from the full retarded solution of the wave equation is singular on the particle world line and hence, it needs to be regularized. To that end, we can use the **Mode-Sum** scheme, which is applied to a harmonic expansion of the gradient of the scalar field:

$$e_{(\alpha)}^\mu \nabla_\mu \Phi = \sum_{lm} \Phi_{(\alpha)}^{lm}(t, r) Y^{lm}(\theta, \varphi)$$

Each mode is finite at the particle location, but their sum diverges. In the mode-sum scheme we can split each mode in a singular piece, $\Phi_{(\alpha)}^S$, which produces no force on the particle, and a regular piece, $\Phi_{(\alpha)}^R$, which is solely responsible for the self-force. The singular part can be expanded as follows:

$$\Phi_{(\alpha)}^S = A_\alpha \left(l + \frac{1}{2} \right) + B_\alpha + \frac{C_\alpha}{l + \frac{1}{2}} + \frac{D_\alpha}{(l - \frac{1}{2})(l + \frac{3}{2})} + \dots$$

where the A , B , C and D coefficients are independent of l . Then, we can compute the self-force by subtracting the singular piece from the full solution:

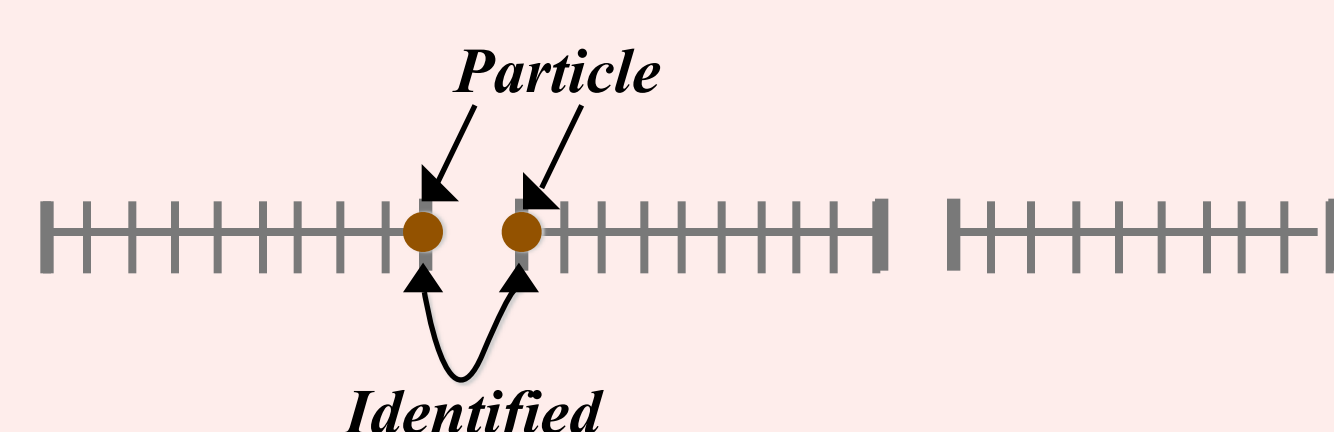
$$\Phi = \Phi^S + \Phi^R \Rightarrow F_\alpha^R := q(\nabla_\alpha \Phi - \nabla_\alpha \Phi^S)$$

Description of the Simulations

In order to compute the self-force, we need to compute first the scalar field generated by the orbiting particle.

We have used the PSC method to solve the equations in the case of a circular orbit. The main challenge in the computations is the treatment of the singular terms generated by the particle. Our computational framework has the following characteristics:

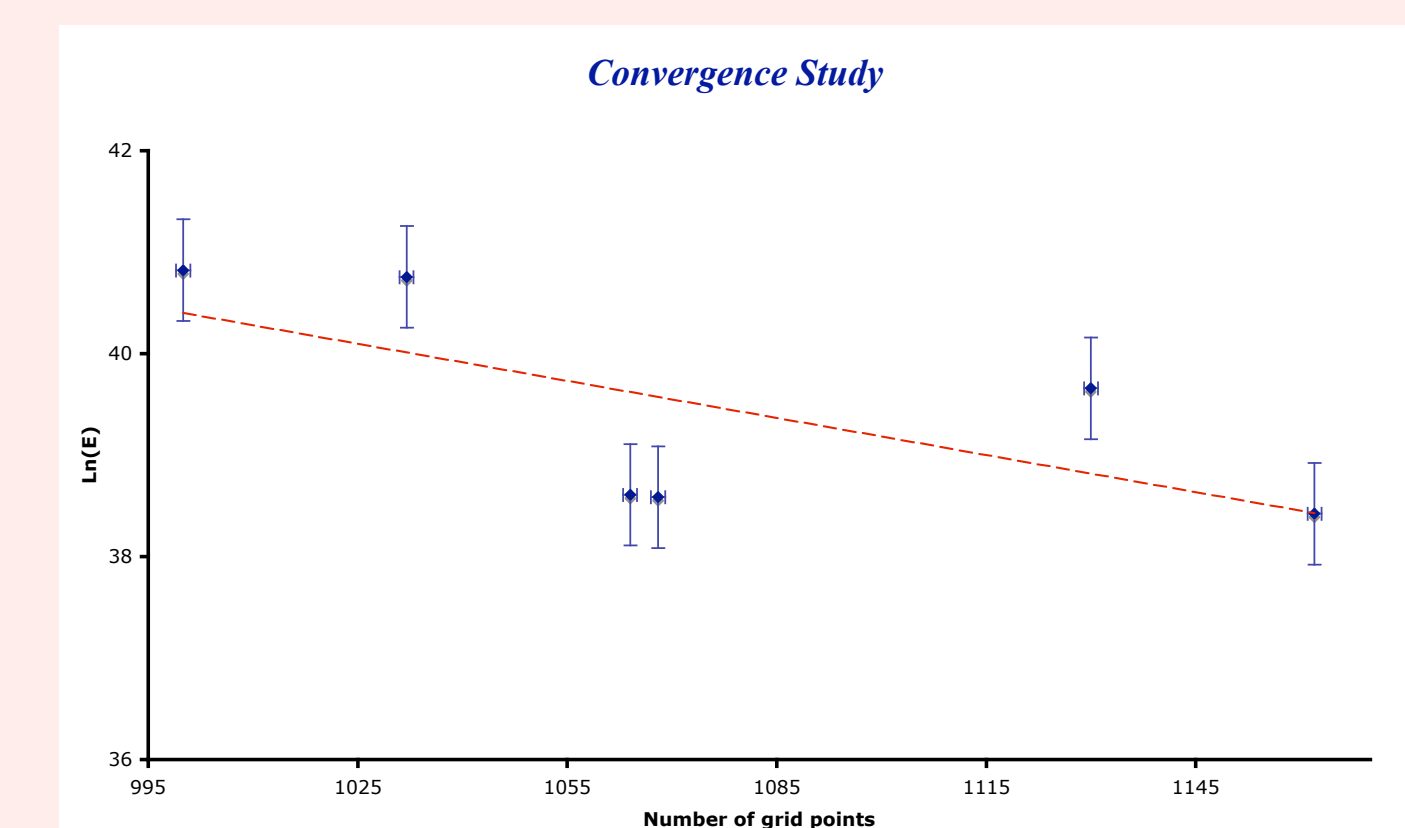
- ◆ We use the PSC method for the spatial discretization and a traditional (finite-differences) Runge-Kutta method for the time evolution.
- ◆ The one-dimensional spatial domain is divided into different disjoint regions. The particle is located at the interface between two such regions (see figure below).
- ◆ We solve the wave-like equations at each region independently. As a consequence, they do not contain singular source terms.
- ◆ We communicate the domains by applying the matching conditions determined by the equations, which account for the presence of the particle.



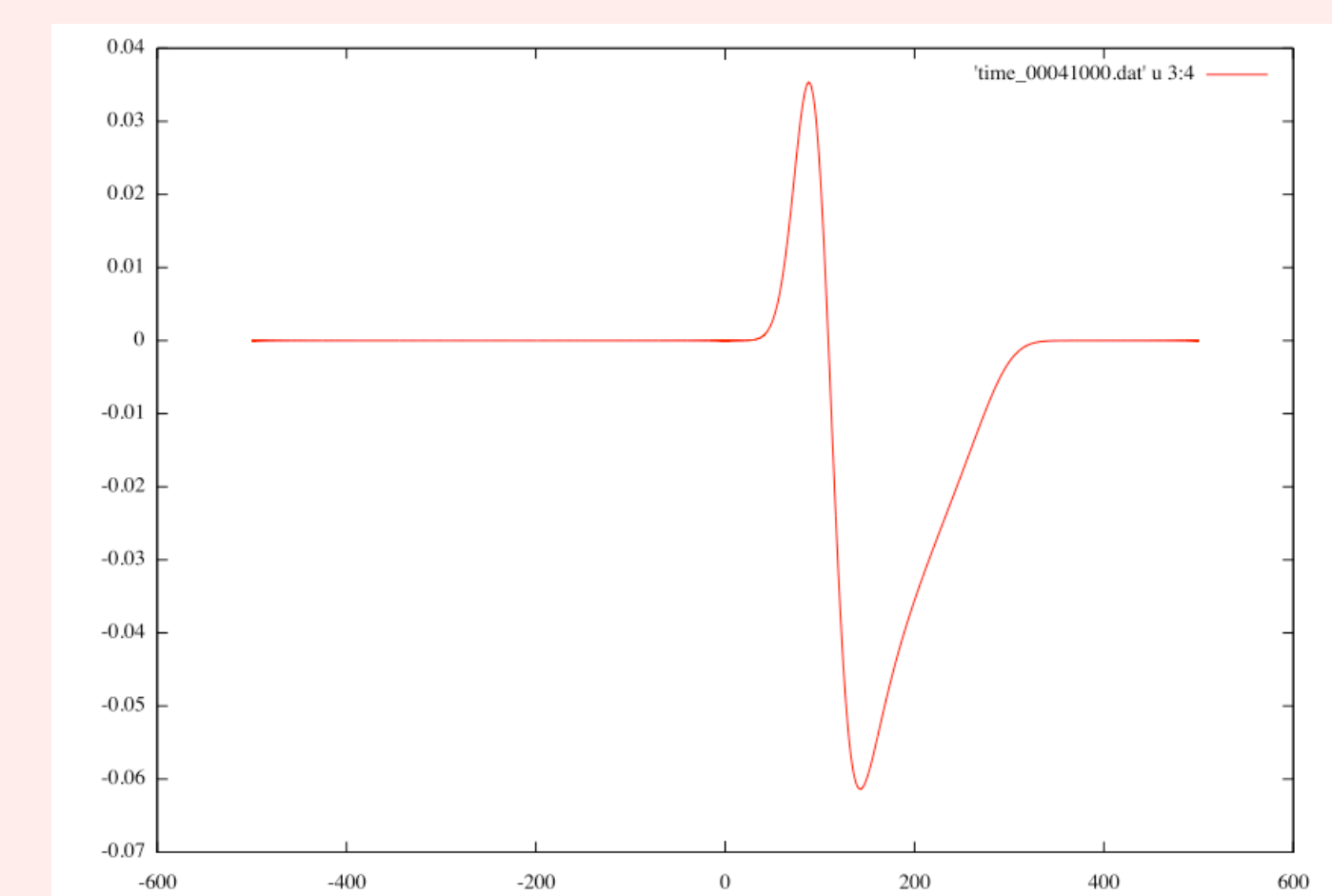
Results from the Simulations

In this simulations we have used two computational domains:

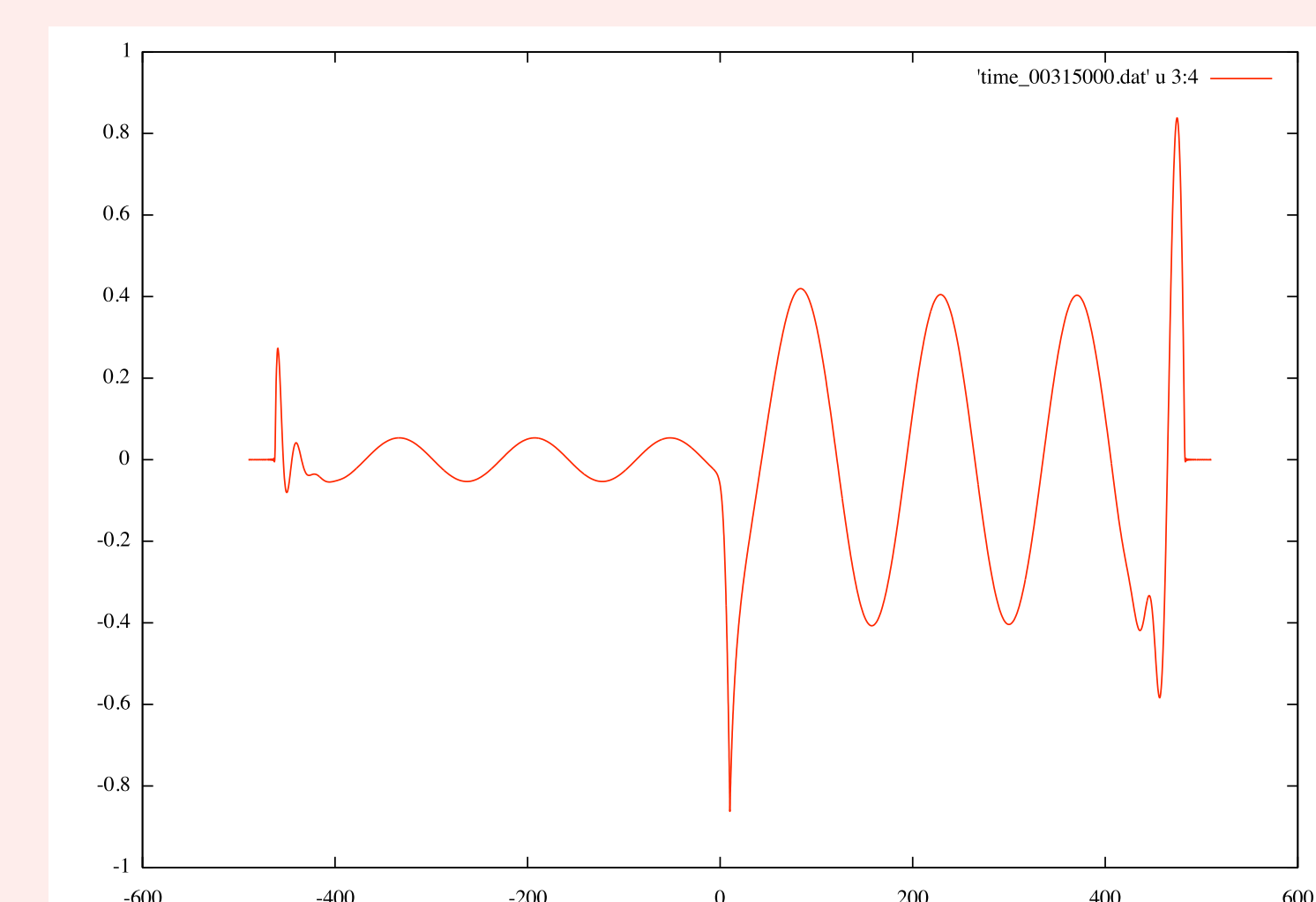
- ◆ Convergence test for the evolution of a gaussian wave packet traveling in flat space. The truncation error decays as e^{-N} .



- ◆ Snapshot of the evolution of a gaussian wave packet scattered by the black hole potential. Case without particle



- ◆ Scalar gravitational waves generated by the evolution of a charged particle orbiting a non rotating black hole.



Future Work

- ◆ Extension of these techniques to general orbits and to the gravitational case.
- ◆ To perform self-force computations.
- ◆ To transfer these techniques to the case of a spinning (Kerr) Black Hole.

References

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Acknowledgments

We acknowledge financial support from the European Commission under a Marie Curie International Reintegration Grant (MIRG-CT-2007- 205005) and the Spanish Ministry of Science and Education (MEC,ESP 2007 - 61712).